The Quadratic Formula and the Discriminant

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is called the **quadratic formula**. The quadratic formula can be used to factor or solve any polynomial in the form: $ax^2 + bx + c$ where $a \neq 0$. When using the quadratic formula, it is important to remember that there are three different types of answers you can get. The type of answer you will get depends on what the discriminant is in the problem. The **discriminant is b^2 - 4ac**. You may see that it is the part of the quadratic formula that is in the square root. The three types of answers you can get with the quadratic formula are two real solutions, two imaginary solutions, or one real solution. If the discriminant is greater than zero, there will be two real solutions as in examples one and two.

Example 1: $2x^2 + 5x – 3$ Discriminant = 49

$$\frac{-(5) \pm \sqrt{(5)^2 - 4(2)(-3)}}{2(2)} \rightarrow \frac{-5 \pm \sqrt{49}}{4} \rightarrow \frac{-5 \pm 7}{4} \rightarrow \left\{ \begin{array}{l} x = \tfrac{1}{2} \\ x = -3 \end{array} \right\}$$

Example 2:
$x^2 + 5x + 3$ Discriminant = 13

$$\frac{-(5) \pm \sqrt{(5)^2 - 4(1)(3)}}{2(1)} \rightarrow \frac{-5 \pm \sqrt{13}}{2} \rightarrow x = \frac{-5 \pm \sqrt{13}}{2}$$

If the discriminant is less than zero, there will be two imaginary solutions as in example three.

Example 3: $x^2 – 4x + 5$ Discriminant = -4

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} \rightarrow \frac{4 \pm \sqrt{-4}}{2} \rightarrow \frac{4 \pm 2i}{2} \rightarrow \left\{ \begin{array}{l} x = 2 + i \\ x = 2 - i \end{array} \right\}$$

Finally, if the discriminant is equal to zero, then there will be one real solution as in example four.

Example 4: $x^2 – 6x + 9$ Discriminant = 0

$$\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)} \rightarrow \frac{6 \pm \sqrt{0}}{2} \rightarrow \frac{6}{2} \rightarrow \left\{ x = 3 \right\}$$

Sketching a graph:

To sketch a graph of a quadratic equation, you will need to find the vertex of the parabola, y-intercept, and x-intercepts.

To find the vertex, use the formula: $x = \frac{-b}{2a}$

Then use the x-value in the original formula to find the y-value.

To find the y-intercept, set x = 0 and solve for y. The ordered pair will be the y-intercept. To find the x-intercept(s), set y = 0 and solve for x. You may get more than one ordered pair. You can also use the quadratic formula to find the x-intercepts.

To make the sketch, use the points that you found for the vertex, y-intercept, and x-intercept(s) and draw a curve through the points.

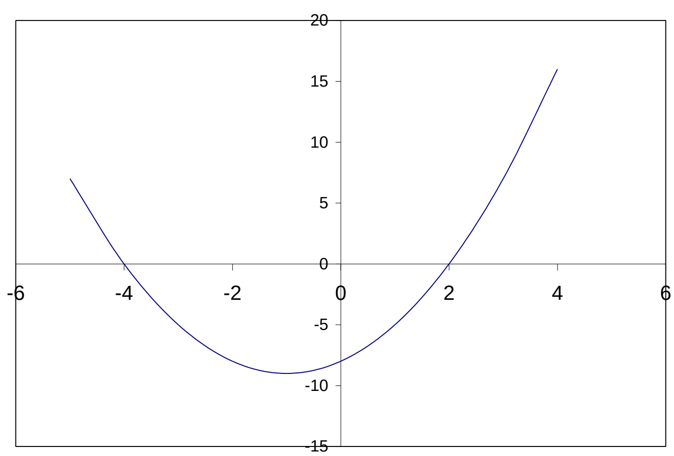
Example 5: $y = x^2 + 2x - 8$

Finding the x-intercept(s)

$y = x^2 + 2x - 8$
$0 = x^2 + 2x - 8$
$0 = (x - 2)(x + 4)$

$0 = x - 2 \qquad 0 = x + 4$
$+2 \quad +2 \qquad -4 \qquad -4$
$2 = x \qquad -4 = x$

So the x-intercepts are (2,0) and (-4,0)



Finding the y-intercept

$y = x^2 + 2x - 8$
$y = 0^2 + 2(0) - 8$
$y = 0 + 0 - 8$
$y = -8$
(0,-8) = y-intercept

Finding the vertex

$x = \frac{-b}{2a}$

$x = \frac{-2}{2 * 1}$

$x = \frac{-2}{2}$

$x = -1$

Now that we know what the x is, we can find the y-value.

$y = x^2 + 2x - 8$
$y = (-1)^2 + 2(-1) - 8$
$y = 1 - 2 - 8$
$y = -9$
Vertex is (-1, -9)

Sample Problems

For each of the following quadratic functions, find the vertex, y-intercept, and the x-intercept(s) of the parabola. Sketch the graph based on this information.

1. $y = 2x^2 – 8x + 6$
2. $y = -x^2 – 4x$
3. $y = (x + 5)(x + 1)$
4. $y = 1/3x^2 - 2/3x - 8/3$
5. $y = -1/4x^2 + 2x - 3$
6. $y = ½x^2 - 2x$
7. $y = -4x^2 + 24x - 32$

Use the quadratic formula to solve each of the following equations.

8. $x^2 - 9x + 20 = 0$
9. $x^2 - 2x - 15 = 0$
10. $x^2 - 7x + 12 = 0$
11. $x^2 + 10x + 24 = 0$
12. $x^2 - 3x - 28 = 0$
13. $-6x^2 + 7x + 20 = 0$
14. $3x^2 - 20x + 12 = 0$
15. $6x^2 - 11x - 10 = 0$
16. $x^2 - 2x - 4 = 0$
17. $x(x - 7) = 2$
18. $x^2 = 10x - 23$
19. $1/x + x/3 = 4$
20. $8x(x - 1) = 1$
21. $4x + 25/x = 20$
22. $5x^2 + 3x + 10$
23. $5x^2 + 4x + 5/4$

Solutions

1. Vertex: (2,-2) y-intercept: (0,2) x-intercepts: (1,0)(3,0)
2. Vertex: (-2,4) y-intercept: (0,0) x-intercepts: (-4,0)(0,0)
3. Vertex: (-3,-4) y-intercept: (0,5) x-intercepts: (-5,0)(-1,0)
4. Vertex: (1,-3) y-intercept: (0,-8/3) x-intercepts: (-2,0)(4,0)
5. Vertex: (4,1) y-intercept: (0,-3) x-intercepts: (2,0)(6,0)
6. Vertex: (2,-2) y-intercept: (0,0) x-intercepts: (0,0)(4,0)
7. Vertex: (3,4) y-intercept: (0,-32) x-intercepts: (2,0)(4,0)
8. x = 4 x = 5
9. x = 5 x = -3
10. x = 4 x = 3
11. x = -6 x = -4
12. x = 7 x = -4
13. x = 5/2 x = -4/3
14. x = 6 x = 2/3
15. x = 5/2 x = -2/3
16. x = $1 \pm \sqrt{3}$
17. x = $\frac{7 \pm \sqrt{57}}{2}$
18. x = $5 \pm \sqrt{2}$
19. x = $6 \pm \sqrt{33}$
20. x = $\frac{8 \pm 7\sqrt{2}}{16}$
21. x = 5/2
22. x = $\frac{-3 \pm i\sqrt{191}}{10}$
23. x = $\frac{-4 \pm 3i}{10}$