MTH 03 – 04 Word Problems

This worksheet is to be used in conjunction with "Algebra: Introductory and Intermediate 3rd Edition"

First Degree Word Problems

Value Mixture

The owner of a delicatessen mixed coffee that costs $4.50 per pound with coffee that costs $3.00 per pound. How many pounds of each were used to make a 20-pound blend that costs $3.90 per pound?

To solve this problem, start by making a grid with 3 columns and 3 rows. The rows will be used for each type of coffee. The bottom row is always the sum of the top two. The columns from left to right are ***Cost***, ***Amount***, and ***Value***. Fill in *cost* and *amount* with the information from the problem. *Value* is always *cost* **times** *amount*. Since the problem does not tell the amount of $4.50 and $3.00 coffees used, select ***x*** as one of their amounts. For this example, ***x*** was selected for the $4.50 coffee. Now to find the amount for the $3.00 coffee: The problem tells us that we have a 20 lb. mixture and ***x*** amount of one of the coffees used to make that mixture. From this, we can determine that the $3.00 coffee's amount must be ***20 –x***. Now all of the *cost* and *amount* columns are filled. Next, **multiply** *cost* by *amount* to find *value*. The first two rows under *value* should be added together to form the bottom row value. Use the information under *value* to write an equation by adding the first two rows and setting them equal to the last row. Solve algebraically.

	Cost	Amount	Value
$4.50 coffee	4.50	x	4.50x
$3.00 coffee	3.00	20 – x	3.00(20 – x)
$3.90 coffee	3.90	20	3.90(20)

$$4.50x + 3.00(20 - x) = 3.90(20)$$
$$4.50x + 60 - 3x = 78$$
$$1.50x + 60 = 78$$
$$-60 \quad -60$$
$$\frac{1.50x}{1.50} = \frac{18}{1.50}$$
$$\mathbf{x = 12}$$

Solution:
x = 12 so there are **12 lb**. of the $4.50 coffee.
For the $3.00 coffee, substitute 12 into its x value: 20 – x
20 – 12 = 8 lbs. of the $3.00 coffee.

Percent Mixture
A coffee blend that is 30% java beans is mixed with a coffee blend that is 55% java beans to make a mixture that is 40% java beans. How many pounds of each blend will be used to make 5 pounds of the 40% java bean mixture?

To solve this problem, start by making another grid with 3 columns and 3 rows. The rows will be used for each type of coffee. The bottom row is always the sum of the top two. The columns from left to right are ***Percent***, ***Amount***, and ***Value***. Fill in the *percent* and *amount* with the information from the problem. *Value* is always *percent* **times** *amount*. Since the problem does not tell the amount of 30% and 55% coffees used, select ***x*** as one of their amounts. For this example, ***x*** was selected for the 30% coffee. Now to find the amount for the 55% coffee: The problem tells us that we have a 5 lb. mixture and ***x*** amount of one of the coffees used to make that mixture. From this, we can determine that the 55% coffee's amount must be **5 –*x***. Now all of the *percent* and *amount* columns are filled. Next, **multiply** *percent* by *amount* to find *value*. The first two rows under *value* should be added together to form the bottom row value. Use the information under *value* to write an equation by adding the first two rows and setting them equal to the last row. Solve algebraically. ****Note**: *When working with percents, convert to decimals.*

	Percent	Amount	Value
30% coffee	0.30	x	0.30x
55% coffee	0.55	5 – x	0.55 (5 – x)
40% coffee	0.40	5	0.40(5)

$0.30x + 0.55(5-x) = 0.40(5)$
$0.30x + 2.75 - 0.55x = 2$
$-0.25x + 2.75 = 2$
$\quad - 2.75 \quad -2.75$
$\frac{-0.25x}{-0.25} = \frac{-.75}{-0.25}$
$x = 3$

Solution:
x = 3, so 3 pounds of the 30% coffee is used in the mixture.
For the amount of the 55% coffee, we plug x = 3 into the amount of the 55% coffee.
5 – x = 5 – 3 = 2
So, **2 pounds of the 55% coffee is used in the mixture.**

Investment

Johnny invests $6,000 at an annual simple interest rate of 14%. How much additional money must he invest at an annual simple interest rate of 10% so that the total interest earned is 12% of the total investment?

When working with investment problems, *Interest* is calculated by multiplying *Principle* **times** *Rate*. Just as before, setup a grid with three columns and three rows. Label the columns as **Principle** (P) which is the amount of money invested, **Rate** (R), and **Interest** (I). Then label the rows as **Account 1**, **Account 2**, and **Total Investment.** When you input the rates, be sure you change the percents to decimals. Once you input the given information into the *Principle* and *Rate* columns, multiply *Principle* times *Rate* to get the *Interest.* The equation you create from the last column is *Account 1 Interest + Account 2 Interest = Total Investment Interest.* Solve algebraically.

****Note:** For Investment problems, the total investment row may or may not be needed depending on the problem. In this problem, it is needed.

	Principle	Rate	Interest
Account 1	$6,000	.14	.14(6,000)
Account 2	x	.10	.10x
Total Investment	**6,000 + x**	.12	.12(6,000 + x)

$$.14(6{,}000) + .10x = .12(6{,}000 + x)$$
$$840 + .10x = 720 + .12x$$
$$-.10x \qquad -.10x$$
$$840 = 720 + .02x$$
$$-720 \quad -720$$
$$\frac{120}{.02} = \frac{.02x}{.02}$$
$$6{,}000 = x$$

$6,000 must be invested in the second account.

Uniform Motion

A family drove to a beach resort at an average speed of 55 mph and later returned over the same road at an average speed of 65 mph. Find the distance to the resort if the total driving time was 12 hours.

For uniform motion problems, use the equation Rate x Time = Distance. So again, you will setup a grid with three columns and three rows. Label the columns as **Rate**, **Time**, and **Distance**. We are given the rates for the two cars. The time is given as a total. If the total time is 12 hours, you can label the time to the resort as **x** and the other car would be the total minus **x**. Multiply each row straight across (rate times time) to get the *Distance* column. To set up the equation you will need to decide the relationship in the resulting *Distance* column. In this case, the first two values under the *Distance* column will be set equal to each other because the distance traveled both ways is the same (equal). Solve algebraically. ****Note:** For distance problems, the bottom row may or may not be needed depending on the problem. In this case, only one cell on the bottom is needed.

	Rate	Time	Distance
To Resort	55 mph	x	55x
From Resort	65 mph	12 – x	65(12 – x)
Total		12	

$55x = 65(12 - x)$
$55x = 780 - 65x$
$+65x \qquad +65x$
$\frac{120x}{120} = \frac{780}{120}$

$x = 6.5$ hours
Now to find the distance to the resort:
Rate * Time = Distance
55 mph * 6.5 hours = **357.5 miles**

The total distance to the resort is 357.5 miles.

A car leaves for its destination traveling 55 mph. A second car leaves for the same destination an hour and 30 minutes later travelling 65 mph. How long will it take the second car to overtake the first car?

You are given information about two separate cars. We will set up the same columns: **Rate**, **Time**, and **Distance**. The rates of each car are given in the problem. The time is given as a relationship between the two cars. If we say that the first car has been traveling for **x** hours, than the second car will be traveling for an hour and a half less than **x** or **x – 1.5** hours. They are traveling the exact same distance and we want to know when they will reach the same point. Again, in the case of this problem the values in the last column will be set equal to each other because the distances will be the same (equal). Solve algebraically.

	Rate	Time	Distance
First Car	55 mph	x	55x
Second Car	65 mph	x - 1.5	65 (x – 1.5)

$55x = 65(x - 1.5)$
$55x = 65x - 97.5$
$-65x \quad -65x$
$\frac{-10x}{-10} = \frac{-97.5}{-10}$
$\mathbf{x = 9.75}$

Solution: The first car is traveling for 9.75 hours. To determine how long it will take the second car to catch up to the first car, we must plug in **9.75** into **x – 1.5.**
9.75 – 1.5 = **8.25 hours** or **8 hrs 15 min**.

Application Problems in Two Variables

A cargo ship traveling with the current traveled 180 miles in 9 hours. On its return voyage traveling against the current, it took 18 hours to travel the same distance. Find the speed of the cargo ship and the current.

Word problems with **two variables** are similar to word problems with one variable. The difference is that you have to write two equations instead of one. First, draw a grid with three columns and two rows. The columns are **Rate, Time,** and **Distance**. Label one of the rows as **With Current** and the other as **Against Current**. Fill in the information that you know from the problem (the time the boat was traveling and the distances traveled by the boat). Since both speeds are unknown, use **x** for the speed of the boat and **y** for the speed of the current. Always use variables to represent the unknown numbers. Under **Rate**, the speed of the boat and the current work together so we add x + y. For the second row, the speed of the boat and the current work against each other so we subtract. Now that the grid is filled, multiple **rate * time** and set the solution equal to the distance (180). This will create two equations. This example uses substation to solve the equation. Any method could be used after the equations are written.

	Rate	Time	Distance
With Current	$x + y$	9	180
Against Current	$x - y$	18	180

$9(x + y) = 180$
$18(x - y) = 180$

$9x + 9y = 180$
$-9x \qquad -9x$
$\frac{9y}{9} = \frac{180 - 9x}{9}$
$y = 20 - x$

$18(x - y) = 180$
$18(x - (20 - x)) = 180$
$18(x - 20 + x) = 180$
$18(2x - 20) = 180$
$36x - 360 = 180$
$+360 \quad +360$
$\frac{36x}{36} = \frac{540}{36}$
$x = 15$

So the **speed of the boat is 15 mph**

$x = 15$
$y = 20 - x$
$y = 20 - 15$
$y = 5$ and the **speed of the current is 5 mph.**

A company sells stress balls and wrist supports. The cost of a stress ball is \$3.75 in materials and \$3.25 in labor. The cost of materials for a wrist support is \$7.50 and the cost of labor is \$4.50. If the company plans on spending \$5313.75 on materials in one day and \$3943.25 in labor, how many stress balls and wrist supports is the company planning on manufacturing?

In this **two variable word problem**, we are looking for the number of stress balls and wrist supports that will be made. Let **x** be the number of stress balls and **y** be the number of wrist supports. Again draw a grid with three columns and two rows. Since the costs for the two items are broken into material cost and labor cost, label one of the rows as **Materials** and the other as **Labor**. Label the columns as **Stress Balls, Wrist Supports,** and **Total Cost**. Now enter the information into the grid. First, enter the costs for the stress balls and wrist supports. Remember to put the corresponding variable in the costs. Then enter the total costs into the grid. Once the grid is filled, form the equations. To do this, add the first columns together and set the answer equal to its total's cost. Each row is a difference equation. This example uses substitution to solve for the equations. Any method could be used after the equations are written.

	Stress Balls	Wrist Supports	Total Costs
Materials	3.75x	7.50y	5313.75
Labor	3.25x	4.50y	3943.25

$3.75x + 7.50y = 5313.75$
$3.25x + 4.50y = 3943.25$

$3.75x + 7.50y = 5313.75$
$\quad -7.50y \quad\quad -7.50y$
$\dfrac{3.75x}{3.75} = \dfrac{5313.75 - 7.50y}{3.75}$
$x = 1417 - 2y$

$3.25x + 4.50y = 3943.25$
$3.25(1417 - 2y) + 4.50y = 3943.25$
$4605.25 - 6.50y + 4.50y = 3943.25$
$4605.25 - 2y = 3943.25$
$-4605.25 \quad\quad -4605.25$
$\dfrac{-2y}{-2} = \dfrac{-662}{-2}$
$y = 331$ So **331 wrist supports** will be made.

$x = 1417 - 2y$
$x = 1417 - 2(331)$
$x = 1417 - 662$
$x = 755$ So **755 stress balls** will be made.

Work and Uniform Motion

Joe can wash a car in 12 minutes. Tom can wash a car in 6 minutes. How long would it take them to wash a car if they work together?

For **work and uniform motion problems**, your grid will have the columns **Rate, Time,** and **Part.** *Rate* is the speed at which the work is done. *Time* is the amount of time it takes to complete the job if working together. *Part* is found by multiplying rate and time together. The rate at which Joe works is 1 car per 12 minutes so the fraction 1/12 goes under Joe's rate. Tom washes 1 car per 6 minutes so the fraction 1/6 goes under Tom's rate. We do not know the amount of time it takes to wash the car when they work together, so an **x** goes under each one's time. Now multiply across to find *Part.* Lastly, add the parts together and set them equal to 1(1 because you want to know how long it takes to wash **one** car). Solve algebraically.

	Rate	Time	Part
Joe	1/12	x	x/12
Tom	1/6	x	x/6

$$\frac{x}{12} + \frac{x}{6} = 1$$

$$\frac{x}{12} + \frac{x}{6}\left(\frac{2}{2}\right) = 1$$

$$\frac{x}{12} + \frac{2x}{12} = 1$$

$$\left(\frac{12}{3}\right)\frac{3x}{12} = 1\left(\frac{12}{3}\right)$$

$$x = \frac{12}{3}$$

$$x = 4$$

So the amount of time it will take is **4 minutes.**

Sample Problems

First-Degree Equations

1. A concession stand worker pours a pound of butter costing $3.60 per pound over three pounds of popcorn costing $0.32 a pound. How much will the buttered popcorn cost per pound?

2. Sam invests $22,500 into two accounts. The first account has an annual simple interest rate of 1.5%. The other account has an annual simple interest rate of 3%. If Sam receives the same amount of interest from each account, how much did he invest in each account?

3. Amy mixes a 15% mercury solution with an 85% mercury solution. How much of each solution should she to make 2.5 liters of a solution that is 50% mercury?

4. Two cyclists start at the same time from opposite ends of a course that is 42 miles long. One cyclist is riding at 16 mph and the second is riding at 12 mph. How long after they began will they meet?

Application Problems in Two Variables

5. JD flies to an airport and then back. He was able to fly to the airport in five hours traveling with the wind. On the way back, he flew against the wind, which took him seven and a half hours. If the distance from airport to airport is 1,200 miles, how fast was the airplane flying and the wind blowing?

6. A Dairy Factory makes cheese and ice cream. A unit of cheese costs $0.25 in ingredients and $1.50 in labor. A unit of ice cream costs $2.00 in ingredients and $1.25 in labor. If the factory plan on spending $165.75 for ingredients and $275.00 for labor, how many units of cheese and ice cream is the factory planning on making?

7. Donna purchased 37 lemons and 2 bags of sugar to make lemonade. The total cost was 15.65. Later that week she purchased (at the same prices) 51 lemons and 3 bags of sugar for a total of $21.90. Find the prices of the lemons and bags of sugar.

Work and Uniform Motion Problems

8. A small pump can fill a swimming pool in 75 minutes and a large pump can fill the same swimming pool in 15 minutes. How long would it take both pumps working together to fill the swimming pool?

9. Ann can give a dog a bath in 15 minutes. When Ann works with Sara's help it takes 10 minutes to give a dog a both. How long would it take Sara to give a dog a bath working alone?

10. One machine runs 8 times faster than an older machine. When the two machines work together it takes them 20 hours to complete a job. How long does it take the older machine to complete the same job working by itself?

11. On the way home, Sue traveled 35 miles at a constant rate through congested traffic. Then she traveled 30 miles per hour faster for an additional 12.5 miles on an expressway. If the entire trip took 2 hours, how fast was she traveling in the congested traffic and on the expressway?

Solutions

1. $1.14 a pound
2. $7,500 in the 1.5% account and $15,000 in the 3% account
3. She should use 1.25 liters of the 15% solution and 1.25 liters of the 85% solution
4. 1.5 hours
5. The airplane is traveling at 200 mph and the wind is traveling at 40 mph.
6. 75 units of cheese and 100 units of ice cream
7. A lemon costs $0.35 and a bag of sugar costs $1.35
8. 12.5 minutes
9. It would take Sara 30 minutes to give a dog a bath working alone
10. 180 hours
11. 20 mph in congested traffic and 50 mph on the expressway