

Linear Equations

Domain and Range

Domain refers to the set of possible values of the x -component of a point in the form (x,y) . **Range** refers to the set of possible values of the y -component of a point in the form (x,y) . If you are asked to find the domain of a set of points, simply list the x -values of those points. Likewise, if you are asked to find the range of a set of points, simply list the y -values of those points.

Example 1: Find the domain and the range of the following set.

$$\{(4,5), (7,8), (-1,3), (3,3), (2,-3)\}$$

$$\text{Domain: } \{-1, 2, 3, 4, 7\}$$

$$\text{Range: } \{-3, 3, 5, 8\}$$

What is a Function?

An equation or grouping of ordered pairs (or points) is a function if and only if no two ordered pairs have the same first coordinate and different second coordinates.

Example 2: Is $\{(4,5), (2, -4), (1,3)\}$ a function?

Yes, it is a function. None of the x -values repeat.

Example 3: Is $\{(1,-4), (3,5), (3,4), (4, 5)\}$ a function?

No, it is not a function. $(3,5)$ and $(3,4)$ have the same x -value and different y -values.

Example 4: Is $\{(-1,-2), (2,3), (3,7), (4, 10), (2,3)\}$ a function?

Yes, it is. $(2,3)$ is just repeated.

Functional Notation $f(x)$

$f(x)$ is a notation for the naming of functions. The letter f is the name of the function and (x) represents the variable in the function. For example, $f(3)$ means that you should replace the x 's with the number 3.

Example 5: Given $f(x) = x^2 + 5$, find $f(-2)$.

$$f(x) = x^2 + 5$$

$$f(-2) = (-2)^2 + 5$$

$$f(-2) = 4 + 5$$

$$f(-2) = 9$$

Example 6: Evaluate $D(z) = 3z - 1$, where $z = 2$.

$$D(2) = 3(2) - 1$$

$$D(2) = 6 - 1$$

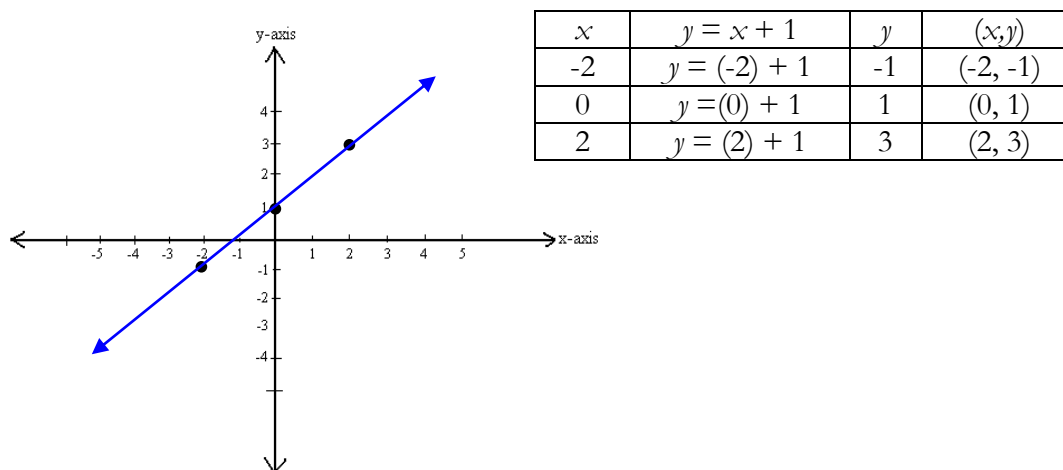
$$D(2) = 5$$

Graph by Plotting Points

Linear functions are straight lines. We can see that an equation will create a straight line when the exponent on each variable is 1. In order to graph a straight line, a minimum of two points is required; however, it is recommended that at least three points are found in order to ensure accuracy.

To find points from a given function or equation, simply choose values for the x coordinate and plug the values into the equation in order to find the corresponding y values. Many times a chart is used to find these values. Once the ordered pairs have been found, they can be plotted on the rectangular coordinate system.

Example 7: Graph $y = x + 1$



There are two forms that are used with linear equations:

Standard form: $Ax + By = C$, where A , B , and C are real numbers

Slope-intercept form: $y = mx + b$, where $m = \text{slope}$ and $b = \text{y-intercept}$

These will be discussed in more detail in the following pages. It is important to remember that you can switch between the two forms just by algebraically rearranging a problem.

Example 8: Write $2x + 3y = 12$ in the form $y = mx + b$

$$2x + 3y = 12$$

$$-2x \quad -2x$$

$$\frac{3y}{3} = \frac{-2x + 12}{3}$$

$$y = (-2/3)x + 4 \quad \text{the equation is now in slope-intercept form}$$

Standard Form $Ax + By = C$

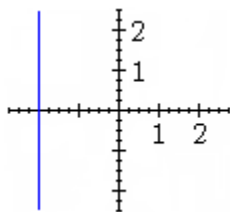
This is one of the two forms of a linear equation. The letters A , B , and C represent numbers. The numbers may not be fractions. The most common way to graph an equation in standard form is to find the x and y intercepts. The x and y intercepts are the two points where the line crosses the x and y axis. To find the intercepts, first set x equal to 0 and solve for y (this is called the y -intercept). Then set y equal to 0 and solve for x (this is called the x -intercept).

Example 9: Find the intercepts of $2x + 4y = 12$.

| | |
|-------------------------------|-------------------------------|
| y -intercept | x -intercept |
| $2(0) + 4y = 12$ | $2x + 4(0) = 12$ |
| $0 + 4y = 12$ | $2x + 0 = 12$ |
| $\frac{4y}{4} = \frac{12}{4}$ | $\frac{2x}{2} = \frac{12}{2}$ |
| $y = 3$ | $x = 6$ |
| $(0,3)$ | $(6,0)$ |

The x -intercept = $(6,0)$, and the y -intercept = $(0,3)$.

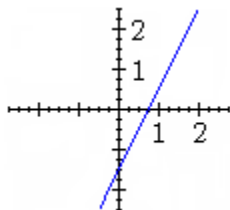
Example 10: Find the intercepts of $x = -2$, then graph it.



| | |
|-------------------|----------------|
| y -intercept | x -intercept |
| $0 = -2$ | $x = -2$ |
| No y -intercept | $(-2,0)$ |

This is a special case with a linear equation. There is no y in the equation. This means that the line does not pass through the y -axis, so this will be a vertical line going through $(-2,0)$.

Example 11: Graph $4x - 2y = 3$.



| | | | | | | | | |
|-----------------|-----------------|--|-----|-----|---|--------|-------|---|
| $4(0) - 2y = 3$ | $4x - 2(0) = 3$ | <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 5px;">x</td> <td style="padding: 5px;">y</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">0</td> <td style="padding: 5px;">$-3/2$</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">$3/4$</td> <td style="padding: 5px;">0</td> </tr> </table> | x | y | 0 | $-3/2$ | $3/4$ | 0 |
| x | y | | | | | | | |
| 0 | $-3/2$ | | | | | | | |
| $3/4$ | 0 | | | | | | | |
| $0 - 2y = 3$ | $4x - 0 = 3$ | | | | | | | |
| $-2y = 3$ | $4x = 3$ | | | | | | | |
| $y = -3/2$ | $x = 3/4$ | | | | | | | |

The y -intercept is $(0, -3/2)$, and the x -intercept is $(3/4, 0)$.

Slope Equation $m = \frac{y_2 - y_1}{x_2 - x_1}$

This formula allows you to find the slope of a line if you know two of the points on the line. The letter m represents the slope. y_2 and y_1 represent the y -values of the points. x_2 and x_1 represent the x -values of the points. The slope of a linear equation can always be written as a fraction! Remember: whole numbers can be written as a fraction by placing the number over a denominator of 1. For example, the whole number 2 is the same as the fraction $2/1$.

Example 12: What is the slope of the linear equation that passes through (1,3) and (-3,4)?

$$\begin{array}{l} x_1 = 1 \quad x_2 = -3 \\ y_1 = 3 \quad y_2 = 4 \end{array} \quad m = \frac{4 - 3}{-3 - 1} = \frac{1}{-4} = -1/4$$

Example 13: What is the slope of the linear equation that passes through (-2,9) and (5,-1)?

$$\begin{array}{l} x_1 = -2 \quad x_2 = 5 \\ y_1 = 9 \quad y_2 = -1 \end{array} \quad m = \frac{-1 - 9}{5 - -2} = \frac{-10}{7}$$

Slope-Intercept Form $y = mx + b$

Slope-intercept form is the second form of a linear equation. The letters m and b represent the slope and the y -intercept. With this information you can graph the linear equation. You want to start by plotting the y -intercept. First, plot the point $(0, b)$ on the y -axis. Then use the slope to find a second point on the line. The slope of the line is rise over the run, or the number of units up over the number of units to the right that you need to move from the point $(0, b)$. This is why the slope is always a fraction. A slope of 6 for example is the same as the fraction $6/1$, so move six units up from the point and one unit to the right. With a slope of 6, you may also move six units down and one unit to the left.

Example 14: Find the slope and the y -intercept of the line $y = 7x - 8$.

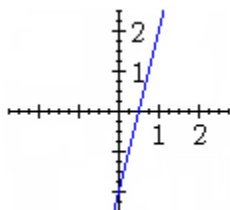
Remember it is in the form $y = mx + b$.

$$m = 7 \quad b = -8$$

So the slope is 7 and the y -intercept is $(0, -8)$.

Example 15: Graph $y = 4x - 2$.

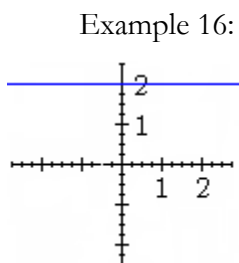
$$m = 4 \quad b = -2$$



Start by graphing the y -intercept $(0, -2)$.

Use the slope, which is rise over run to move up 4 and over 1 from the y -intercept.

Remember that 4 is the same as $4/1$.



Example 16: Graph $y = 2$

There is no x , which means that the slope is 0. 2 is the b value. Therefore, the line goes through $(0,2)$ so put a dot at that point. The slope is 0, which is the same as $0/1$, so we will go up zero and over one. This will be a horizontal line.

Point-Slope Equation $y - y_1 = m(x - x_1)$

The Point-Slope Equation is used to find the equation of a line when you are given either two points on the line or one point on the line and the slope of the line.

One Point and the Slope

If you are given the slope of the line and one point on the line, you will need to follow three steps. First, replace m with the slope of the line, replace x_1 with the x -value of the point, and replace y_1 with the y -value of the point. Second, you will solve for y .

Example 17: Find the equation of the line with a slope of $\frac{1}{2}$ passing through $(2,5)$.

$$\begin{aligned} \text{Slope} &= \frac{1}{2} & \text{Point: } &(2,5) \\ m &= \frac{1}{2} & x_1 &= 2 & y_1 &= 5 \\ y - y_1 &= m(x - x_1) \\ y - 5 &= \frac{1}{2}(x - 2) \\ y - 5 &= \frac{1}{2}x - 1 \\ y &= \frac{1}{2}x + 4 \end{aligned}$$

Two Points

If you are given two points on the line, you will first need to find the slope of the line. (See Slope Equation) Choose one of the points (it does not matter which one), and use that point and the slope to find the equation of the line.

Example 18: Find the equation of the line passing through $(1,5)$ and $(3,2)$.

For this problem you will first need to find the slope of the line.

$$\begin{aligned} x_1 &= 1 & x_2 &= 3 \\ y_1 &= 5 & y_2 &= 2 \end{aligned} \qquad m = \frac{2 - 5}{3 - 1} = \frac{-3}{2}$$

$$\begin{aligned} m &= -3/2 \quad (1,5) & \text{or} & \quad m &= -3/2 \quad (3,2) \\ y - 5 &= -3/2(x - 1) & & & y - 2 &= -3/2(x - 3) \\ y - 5 &= -3x/2 + 3/2 & & & y - 2 &= -3x/2 + 9/2 \\ y &= (-3/2)x + 13/2 & & & y &= (-3/2)x + 13/2 \end{aligned}$$

Parallel and Perpendicular Slopes

If two lines are parallel, they have the same slope. If two lines are perpendicular, they have slopes that are negative reciprocals of each other. Two fractions are negative reciprocals of each other when one is positive and the other is negative and if one reciprocal can be flipped to get the other reciprocal. See Example 17.

Example 19: The following are examples of negative reciprocals.

$$\frac{11}{6} \leftrightarrow \frac{-6}{11} \quad \frac{-3}{1} \leftrightarrow \frac{1}{3} \quad \frac{2}{9} \leftrightarrow \frac{-9}{2}$$

Example 20: Are the lines $y = -2x - 3$ and $y = \frac{1}{2}x + 25$ parallel, perpendicular, or neither?

$$y = -2x - 3 \qquad y = \frac{1}{2}x + 25$$

$$m = -2 \qquad m = \frac{1}{2}$$

The slopes are negative reciprocals so the lines are perpendicular.

Example 21: Are the lines $2x + 4y = 3$ and $3x - 5y = 10$ parallel, perpendicular, or neither?

$$2x + 4y = 3 \qquad 3x - 5y = 10$$

$$4y = -2x + 3 \qquad -5y = -3x + 10$$

$$y = -\frac{1}{2}x + \frac{3}{4} \qquad y = \frac{3}{5}x - 2$$

$$m = -\frac{1}{2} \qquad m = \frac{3}{5}$$

The slopes are not negative reciprocals nor the same, so the lines are neither.

Example 22: Find the equation of the line that passes through (3,21) and is parallel to $y = \frac{1}{4}x - 8$.

$$y = \frac{1}{4}x - 8$$

$$m = \frac{1}{4}$$

The line we want to find will also have a slope of $\frac{1}{4}$ because the lines are parallel.

$$m = \frac{1}{4} \qquad x_1 = 3 \qquad y_1 = 21$$

$$y - y_1 = m(x - x_1)$$

$$y - 21 = \frac{1}{4}(x - 3)$$

$$y - 21 = \frac{1}{4}x - \frac{3}{4}$$

$$y = \frac{1}{4}x + 8\frac{1}{4}$$

Sample Problems

State the domain and range for each. Is the relation a function?

1. $\{(9,-8), (3,-8), (-5,-8)\}$
2. $\{(-4,8), (9,-1), (-3,-1), (-8,5)\}$
3. $\{(8,-8), (4,-5), (0,-8), (8,-1)\}$

Complete

4. $f(x) = -5x^2 - 6x + 12$ find $f(-7)$
5. $f(x) = \frac{-3 + 3x - 6x^2}{4x - 5x^2 - 14}$ find $f(10)$
6. $f(x) = -7/4$ find $f(-2)$

Write the slope of the line that passes through the two points.

7. $(-3, 7)$ $(6,6)$
8. $(-3,5)$ $(-7,3)$
9. $(5,-8)$ $(0,4)$

Find the intercepts of the line.

10. $y = -5$
11. $4x + y = 16$
12. $y = x - 2$

Write an equation of the line that contains the given point and has the given slope.

13. $(-5,-6)$, slope = -3
14. $(-8,-5)$, slope = 5
15. $(2, -7)$, slope = -2

Graph each equation.

16. $y = x$
17. $2x - 2y = -3$

Write an equation of the line that passes through the two given points.

18. $(-4,1)$ $(-2,7)$
19. $(5,5)$ $(11,17)$
20. $(-2,3)$ $(13,63)$

Determine whether the lines are parallel, perpendicular, or neither.

21. $x - 2y = -59$ $2x + y = -63$
22. $-10y = -96 - 22x$ $11x - 5y = -48$
23. $17x + 8y = -47$ $y = -23$

Find the slope and the y -intercept of the line.

24. $10x = -132 - 26y$

25. $4x - y = 8$

26. $y = 11/3x + 18$

Write the slope-intercept form of the equation.

27. $3x + 2y = 2$

28. $4x + y = 26$

29. $-26y = -50 - 10x$

Graph the following.

30. A line that passes through the point (2,1) and has a slope of 1.

31. A line that passes through the point (-5,0) and is parallel to another line with a slope of $4/3$.

Write the equation of the line.

32. y -intercept = 29 x -intercept = $9 \frac{2}{3}$

33. y -intercept = -12 x -intercept = -6

Complete the following.

34. Write an equation of a line that passes through the point (-1,3) and is perpendicular to the following equation: $2x + 4y = -1$.

35. Write an equation of the line that passes through the point (-2,-3) and is perpendicular to the following equation: $y = -\frac{1}{2}x - 3$.

36. Write an equation of the line that passes through the point (3,2) and is parallel to the following equation: $3x + y = -3$.

Solutions

- | | | |
|--------------------------|-------------------|----------------------|
| 1. Domain: 9, 3, -5 | Range: -8 | It is a function |
| 2. Domain: -4, 9, -3, -8 | Range: 8, -1, 5 | It is a function |
| 3. Domain: 8, 4, 0 | Range: -8, -5, -1 | It is not a function |

4. -191
5. $573/474$
6. $-7/4$
7. $-1/9$
8. $1/2$
9. $-12/5$
10. (0, -5)
11. (0, 16), (4, 0)
12. (0, -2), (2, 0)
13. $y = -3x - 21$
14. $y = 5x + 35$
15. $y = -2x - 3$

18. $y = 3x + 13$
19. $y = 2x - 5$
20. $y = 4x + 11$
21. perpendicular
22. parallel
23. neither perpendicular nor parallel
24. $m = -5/13$ $b = -66/13$
25. $m = 4$ $b = -8$
26. $m = 11/3$ $b = 18$
27. $y = -3/2x + 1$
28. $y = -4x + 26$
29. $y = 5/13x + 25/13$

32. $y = -3x + 29$
33. $y = -2x - 12$
34. $y = 2x + 5$
35. $y = 2x + 1$
36. $y = -3x + 11$

