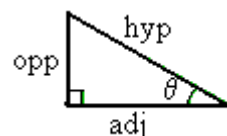


## Common Trigonometric Angle Measurements



$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}} \quad \csc \theta = \frac{\text{hyp}}{\text{opp}} \quad \sec \theta = \frac{\text{hyp}}{\text{adj}} \quad \cot \theta = \frac{\text{adj}}{\text{opp}}$$

$$\text{Hypotenuse} = \sqrt{\text{adjacent}^2 + \text{opposite}^2}$$

Angle in Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
Angle in Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$2\pi$
SIN	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
COS	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
TAN	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
CSC	ND	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	ND	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	ND
SEC	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	ND	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	ND	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1
COT	ND	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	ND	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	ND

Angles	Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>
sin and csc	+	+	-	-
tan and cot	+	-	+	-
cos and sec	+	-	-	+

Reference Angles  $\theta'$

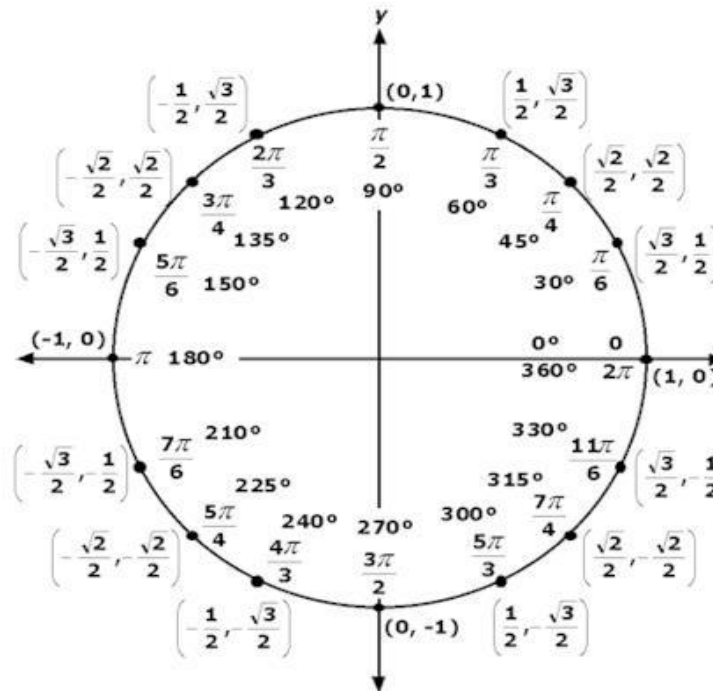
If.....  $0^\circ \leq \theta \leq 90^\circ$  then.....  $\theta' = \theta$   
 $90^\circ < \theta \leq 180^\circ$   $\theta' = 180^\circ - \theta$   
 $180^\circ < \theta \leq 270^\circ$   $\theta' = \theta - 180^\circ$   
 $270^\circ < \theta \leq 360^\circ$   $\theta' = 360^\circ - \theta$

To convert from Radians to Degrees.....

Multiply by  $\frac{180^\circ}{\pi \text{ radians}}$

To convert from Degrees to Radians.....

Multiply by  $\frac{\pi \text{ radians}}{180^\circ}$



When using the unit circle:

Start at  $0^\circ$  and then....

Positive angles move counter clockwise.

Negative angles move clockwise.



### Fundamental Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta} \quad \csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1 \quad 1 + \tan^2 \theta = \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta$$

### Sum and Difference Identities

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

### Double-Angle Identities

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 1 - 2\sin^2 \alpha = 2\cos^2 \alpha - 1$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

### Half-Angle Identities

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan \frac{\alpha}{2} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

### Even and Odd Identities

$$\sin(-\alpha) = -\sin \alpha \quad \cos(-\alpha) = \cos \alpha \quad \tan(-\alpha) = -\tan \alpha \quad \csc(-\alpha) = -\csc \alpha \quad \sec(-\alpha) = \sec \alpha \quad \cot(-\alpha) = -\cot \alpha$$

### Power-Reducing Identities

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2} \quad \cos^2 \alpha = \frac{1 + \cos 2\alpha}{2} \quad \tan^2 \alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}$$

### Product-to-Sum Identities

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)] \quad \cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)] \quad \sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

### Sum-to-Product Identities

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} \quad \sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} \quad \cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

For sums of the form  $a \sin x + b \cos x$  .....

$$a \sin x + b \cos x = k \sin(x + \alpha)$$

$$\text{Where } k = \sqrt{a^2 + b^2}, \quad \sin \alpha = \frac{b}{\sqrt{a^2 + b^2}}, \quad \text{and } \cos \alpha = \frac{a}{\sqrt{a^2 + b^2}}$$



### Arc Length Formula

$$s = r \theta \quad \text{or} \quad \theta = \frac{s}{r} \text{ radians}$$

s = length of the arc

r = radius of the circle

$\theta$  = non-negative radian measure of the central angle

### Converting from Degrees to Minutes to Seconds

$$1^\circ \text{ (degree)} = 60' \text{ (minutes)}$$

$$1' \text{ (minute)} = 60'' \text{ (seconds)}$$

$$1^\circ \text{ (degree)} = 3,600'' \text{ (seconds)}$$

### Linear Speed (v)

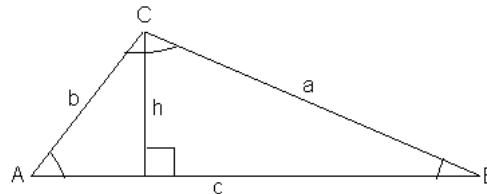
$$v = \frac{s}{t} \quad \begin{array}{l} s = \text{distance traveled} \\ t = \text{time} \end{array}$$

### Angular Speed ( $\omega$ )

$$\omega = \frac{\theta}{t} \quad \begin{array}{l} \theta = \text{measure of angle in radians} \\ t = \text{time} \end{array}$$

### Linear Speed – Angular Speed Relationship

$$v = r\omega \quad \begin{array}{l} v = \text{linear speed} \\ r = \text{radius} \\ \omega = \text{angular speed} \end{array}$$



**Law of Sines:** used to solve triangles when two angles and a side are given or when two sides and an opposite angle are given.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

**Law of Cosines:** used to solve triangles when two sides and the included angle or three sides of the triangle are given.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

**Area (K)**

$$K = \frac{1}{2} bc \sin A = \frac{b^2 \sin C \sin A}{2 \sin B}$$

$$K = \frac{1}{2} ab \sin C$$

$$K = \frac{1}{2} ac \sin B$$

**Area (K) using Heron's Formula**

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$